

Towards the lattice study of M-theory

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We propose the Wilson discretization of the supersymmetric Yang-Mills Quantum Mechanics as a lattice version of the matrix model of M-theory. An $SU(2)$ model is studied numerically in the quenched approximation for $D=4$. A clear signal for the existence of two different phases is found and the continuum pseudocritical temperature is determined. We have also extracted the continuum limit of the total size of the system in both phases and for different temperatures.

1. *Introduction.* Since the discovery of the web of dualities linking different superstring theories, it has been suggested that all these theories are just different perturbative expansions of a single 11-dimensional theory christened M-theory. The true nature of this theory remains, however, mysterious. In [1] Banks, Fischler, Susskind and Shenker proposed a concrete formulation for the degrees of freedom of M-theory in a flat 11-dimensional background.

In this contribution we would like to report on the work done in [2] on a lattice formulation of the BFSS matrix model, thus allowing for a nonperturbative treatment. We use the version with euclidean signature suitable for studying thermodynamical properties of the theory. In fact it turns out that there is a very rich spectrum of physics involved. A study of these properties forms the main motivation for our investigation. Below we briefly review the physics involved as it also provides an incentive for further study.

The BFSS lagrangian for finite N is a dimensional reduction of 10D $SU(N)$ SYM down to 1 dimension, schematically

$$S = \int dt \left(\frac{1}{2} (D_t X^i)^2 + [X^i, X^j]^2 + fermions \right). \quad (1)$$

Equivalently it describes a system of N D0 branes in the decoupling limit. All the dynamics at finite temperature T can be expressed in terms of a dimensionless coupling constant $g_{eff}^2 = g_{YM}^2 N / T^3$. At high temperatures the system is

perturbative. As we lower the temperature we enter the strong coupling regime.

At this stage we may apply the framework of the AdS/CFT correspondence (see [3,4] in this context). An equivalent description is provided by a 10 dimensional Black Hole (BH) supergravity solution, whose Hawking temperature is identified with the temperature of the SYM quantum mechanics (1). In particular the Bekenstein entropy of the black hole calculated from the area of the horizon should be identified with the entropy of the SYM quantum mechanics (for recent new results see [8]) which thus gives a microscopic realization of the ‘macroscopic’ BH entropy. It would be very interesting to directly observe this prediction by studying numerically (1).

Another interesting point is the nature of the transition from the perturbative to the strong coupling phase. In [9] it has been identified with the Horowitz-Polchinsky correspondence point between a black hole phase and ‘excited string’ phase. It’s detailed nature is still unknown.

As one lowers the temperature, one reaches a regime where the string coupling becomes strong and one has to lift the 10D BH solution to a 11D ‘black wave’. Further down on the temperature scale the 11D ‘black wave’ localizes to a 11D BH.

All the above physics should be contained in the strong coupled regime of (1). For most of these questions supersymmetry is crucial. In our exploratory study we perform quenched simulations and focus on showing the existence of two phases in the high and low temperature regimes. When performing simulations in the full

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unquenched model the observed phases should correspond to the black hole phase and the perturbative phase. In the following we present the lattice formulation, give details on the algorithms and present results of first exploratory quenched simulations for $N = 2$.

2. *Lattice formulation.* A direct Monte-Carlo simulation of the action (1) is rather nontrivial. The potential for the bosonic fields $[X^i, X^j]^2$ has flat valleys which may be difficult to handle numerically. Furthermore it is essential to impose the Gauss law constraint or equivalently to project onto gauge invariant states. This may be important especially as the supersymmetry algebra in the continuum closes only on the space of gauge invariant states. Nevertheless the nonperturbative studies of the IIB matrix models (SYM reduced to zero dimensions) in this formulation are well advanced [5,6].

In order to overcome the above difficulties, and to use standard, well developed lattice techniques we decided, however, to follow a different route. The action (1) comes from the standard SYM action in $D = 10$, upon assuming that all fields do not depend on spatial coordinates. We may now use the Wilsonian discretization in $D = 10$ and impose the constancy of link variables in the spatial directions. In this manner we obtain at once a D dimensional hypercubic lattice $N_1 \times \dots \times N_D$ reduced in all space directions to $N_i = 1, i = 1 \dots D-1$. Gauge and fermionic variables are assigned to links and sites of the new elongated lattice in the standard manner. The gauge part of the action is

$$S_G = -\beta \sum_{m=1}^{N_t} \sum_{\mu > \nu} \frac{1}{N} \text{Re}(\text{Tr} U_{\mu\nu}(m)), \quad (2)$$

with $\beta = 2N/a^3 g^2$, and $U_{\mu\nu}(m) = U_\nu^\dagger(m) U_\mu^\dagger(m + \nu) U_\nu(m + \mu) U_\mu(m)$, $U_\mu(m) = \exp(ia g A_\mu(am))$, where a denotes the lattice constant and g is the gauge coupling in one dimension. The integer time coordinate along the lattice is m . Periodic boundary conditions $U_\mu(m + \nu) = U_\mu(m)$, $\nu = 1 \dots D-1$, guarantee correct classical continuum limit (1) with $X_i = g A_i$. In this formulation the projection on gauge invariant states is naturally implemented. Adjoint fermions can be in-

N_t	β_{low}	β_{up}
2	1.25	1.5
3	3.5	5.0
4	8.0	16.0
5	15.0	40.0
<i>fit : $\beta_c = \alpha N_t^\gamma$</i>		
χ^2/NDF	α	γ
.55/2	.17 \pm .05	3.02 \pm .33

Table 1

Estimated location of the transition region $\beta_c \in (\beta_{low}, \beta_{up})$ for different lattice sizes N_t and results of the power fit.

cluded as in the lattice studies of SUSYM theory [7,2].

2. *Results.* We choose the distribution of the Polyakov line $P = \frac{1}{N} \text{Tr} \prod_{m=1}^{N_t} U_D(m)$, as an order parameter. Similarly to lattice QCD, symmetric concentration of the eigenvalues around 0 indicates a low temperature phase (which would have here the interpretation of a black hole phase) where $\langle P \rangle \sim 0$, while clustering around ± 1 (for SU(2)) is characteristic of the high temperature (elementary excitations) phase. Our first result

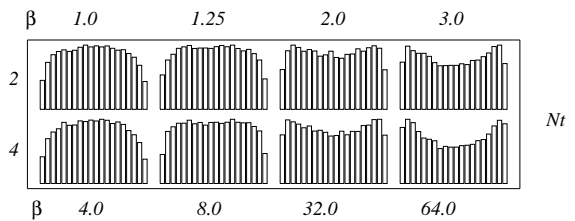


Figure 1. Distribution of the Polyakov line P , $-1 < P < 1$, for different β and N_t .

is evident from Fig. 1. For each N_t a definite change of the shape with β is observed. The system (2) definitely shows the onset of the phase change, even in the quenched approximation and for $N = 2$. Second, the dependence of the pseud-

ocritical temperature β_c on the time extent N_t is consistent with the continuum limit expectations $T_c \sim (g^2 N)^{1/3}$ [8]. Indeed, the temperature of a system is given by $T = 1/(aN_t)$ which implies $\beta_c \sim N_t^3$. This is confirmed in Table 1 where β intervals where the transition occurs are presented for several lattice sizes N_t . Results of the power law fit are also quoted. A good quality of the fit and the agreement with the canonical exponent, $\gamma = 3$, is encouraging. Simultaneously, we obtain the proportionality coefficient α which gives

$$T_c = \left(\frac{\alpha}{2N^2}\right)^{1/3} (g^2 N)^{1/3} = (.28 \pm .03) (g^2 N)^{1/3}. \quad (3)$$

The coefficient in this relation has been determined for the first time. Only proportionality of the two scales was considered [9,4,8]. The pseudocritical temperature and α may depend on N , therefore similar analysis for higher gauge groups is necessary.

Next we study the total size of the system $R^2 = g^2 \sum_a (A_i^a)^2$ [8]. We define for $SU(2)$

$$\langle R^2 \rangle \equiv (4 - \langle (\text{Tr}(U_s))^2 \rangle) / a^2, \quad (4)$$

where U_s is any space link.

We used mostly the standard local Metropolis update with enough thermalization and decorrelation sweeps to take care of the critical slowing down. For example when running at $\tilde{a} = 1.0$ we used 5000 thermalization and 50 decorrelation sweeps, while for $\tilde{a} \equiv ag^{2/3} = 0.1$ about 10^6 thermalization and 5000 decorrelation sweeps were required to achieve independence of the starting configuration. This also agrees with the dynamical exponent $z = 2$. In addition we have developed the new heat bath algorithm designed for an update of the space-space plaquettes in (2) which contain twice the same $SU(2)$ link. In the standard S_3 parametrization of $SU(2)$ distribution of the three vector \vec{u} , $|\vec{u}| = u \leq 1$ is the product of the three dimensional gaussian and the cosh factor which depends on $\sqrt{1-u^2}$. At fixed values of the neighbouring links the center of the gaussian may lie outside the kinematical region $u < 1$, which even for intermediate β makes the simple gaussian generation inefficient. However by a suitable choice of the coordinate system²

²The one with the third axis pointing towards the maxi-

the problem becomes manageable. The quadratic part of the $\log \cosh$ factor is included in the above gaussian and deviations from the exact function are included by the accept-reject step. Results obtained with the new heat bath and the standard Metropolis agree within statistical errors for $\beta < 64$. For higher β the accept-reject step becomes inefficient. For higher β we have also monitored the correlation length in the torelon channel at zero temperature. It shows the canonical scaling with a as expected. Fig. 2 shows the de-

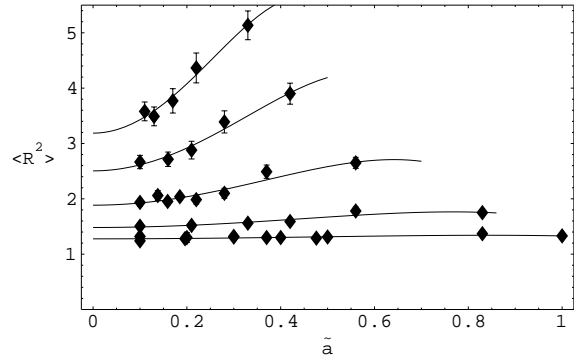


Figure 2. Dependence of the total size of the system on a , for $\tilde{T} \equiv Tg^{-2/3} = .1 - .3, .6, .9, 1.2$ and 1.5 (upwards) in units of $g^{2/3}$. Quartic fits are represented by the solid lines.

pendence of $\langle R^2 \rangle$ (in units of $g^{2/3}$) on \tilde{a} , for several values of the temperature \tilde{T} . MC results depend smoothly on a , at fixed T , confirming the existence of the continuum limit (4). The a dependence is different in low and high temperature regions. For $\tilde{T} > 1.5$ simulations for smaller \tilde{a} are required in order to see the quadratic approach to the continuum. Practically the same results were obtained when we extracted $\langle R^2 \rangle$ from another lattice observable $|\text{Tr}(U_s)|$. Fig. 3 shows the size of a system extrapolated to $a = 0$ as a function of the temperature. Both quadratic and quartic fits of a dependence were used. The stability

mum of a gaussian on a boundary sphere $u = 1$.

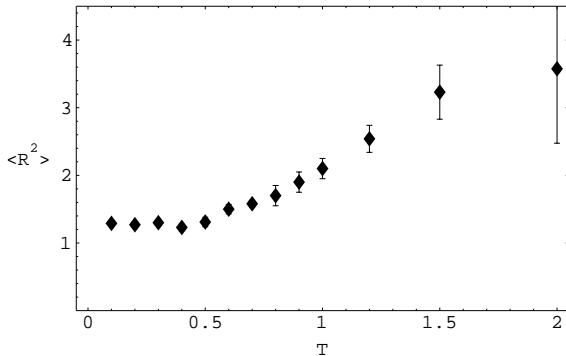


Figure 3. Size of the system (4) extrapolated to the continuum, as a function of the temperature.

of quadratic fits with respect to removing one or two data points with smallest a was also checked. Results of the extrapolation were stable with respect to all these variations. Displayed errors include above systematic effects. The location of the transition region agrees approximately with the estimate (3) of the pseudocritical temperature $\tilde{T}_c = 0.35 \pm 0.04$. Again, it is evident that the system is indeed different in the two regimes. Moreover, our results agree qualitatively with the analytical prediction obtained from a gap equation in the infinite N limit [8]. The latter gives a temperature independent constant at low temperatures and the classical $T^{1/2}$ growth for high temperatures. We have also found a reasonable agreement with a simple mean field model for $SU(2)$ with the gauge projection which will be discussed elsewhere.

3. Outlook. We have constructed, for the first time, a matrix model of M-theory on a lattice. MC simulations show that even the strongly simplified version of the model has different high and low temperature phases as expected in the original theory. Pseudocritical temperature and total size of the system show canonical approach to the continuum limit. This indicates that, at least in this respect, the system may be simpler than the ones encountered in lattice studies of field theories in extended space-time. In particu-

lar, restoration of the supersymmetry, broken by lattice discretization, is feasible.

Of course results presented here are only a hint that we may be on a right track. One urgent task is to repeat present programme for higher N and $D=10$. This, although relatively simple, may require some refinement of existing algorithms or inventing new ones dedicated to the linear systems. Equally important is to include dynamical, supersymmetric fermions. For $D=4$ this is again relatively standard and would allow us to study full intricacies of lattice breaking and subsequent restoration of the supersymmetry in the continuum limit. For $D=10$ the problem of dynamical fermions is open and presents an exciting challenge. Again the linear nature of the system may offer some simplifications.

To conclude, we believe that many interesting applications became available opening new possibilities for the nonperturbative studies of the prototypes of M-theory.

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